8.11) Biaxial Bending
8.12) Load Control Method
8.13) Reciprocal Load Method
8.11) Biaxial Bending

- Methods previously discussed handle bending about only one of the principal axes. In many cases, such as in corner columns or in interior columns with irregular column layout, bending occurs about both principal axes.

- Let \( X \) and \( Y \) denote the directions of the principal axes of the cross section. The 3D sketch is established the usual methods for uniaxial bending, but by drawing \( P_n \) vs. \( M_{ny} \) where the eccentricity is \( e_x \) for bending about the \( Y \) axis only, and \( P_n \) vs. \( M_{nx} \) where the eccentricity is \( e_y \) for bending about the \( X \) axis only. For the case which combines \( X \) and \( Y \) bending, the orientation of the resultant eccentricity is defined by:

\[
\lambda = \tan^{-1}\left(\frac{e_x}{e_y}\right) = \tan^{-1}\left(\frac{M_{ny}}{M_{nx}}\right)
\]
Bending for this case is about an axis defined by the angle $\theta$ with respect to the $X$ axis.

The angle $\lambda$ establishes a plane passing through the vertical $P_n$ axis and making an angle $\lambda$ with $M_{nx}$ axis. This plane represents a *failure surface* in 3-D analogous to the *interaction diagram* in 2-D.

Any combination of $P_u$, $M_{ux}$, and $M_{uy}$ falling inside the failure surface can be applied safely, and any point falling outside the surface represents failure.

Note that the failure surface can be described either by a set of curves defined by radial planes passing through the $P_n$ axis or by a set of curves defined by horizontal plane intersections, each for a constant $P_n$, defining load contours.

Constructing interaction surface can be done by taking successive values of $c$ for a selected angle $\theta$ then using equilibrium equations to find $P_n$, $M_{nx}$, and $M_{ny}$.

The triangular or trapezoidal compression zone is a complication and in general the strain in each reinforcing bar will be different, but these features can be incorporated.

The main difficulty is that the neutral axis will not, in general, be perpendicular to the resultant eccentricity, drawn from the column center to the load $P_n$. For each successive choice of neutral axis, there are unique $P_n$, $M_{nx}$, and $M_{ny}$ and only in special cases the eccentricity becomes perpendicular to the neutral axis chosen. This results in for successive choices of $c$ for any given value of $\theta$, the value of $\lambda$ and $d$ will vary.
8.12) Load Contour Method

- The load contour method is based on representing the failure surface previously discussed by a family of curves corresponding to constant values of \( P_n \) in the form approximated by the non-dimensional interaction equation:

\[
\left( \frac{M_{nx}}{M_{nx0}} \right)^{\alpha_1} + \left( \frac{M_{ny}}{M_{ny0}} \right)^{\alpha_2} \leq 1.0
\]

where:

- \( M_{nx} = P_n e_y \)
- \( M_{nx0} = M_{nx} \) when \( M_{ny} = 0 \)
- \( M_{ny} = P_n e_x \)
- \( M_{ny0} = M_{ny} \) when \( M_{nx} = 0 \)
- \( \alpha_1 \) and \( \alpha_2 \) are exponents depend on column dimensions, amount and distribution of steel reinforcement, steel and concrete properties, cover, and size of lateral ties and spirals.

- Curves shown for \( \alpha_1 = \alpha_2 = \alpha \).
8.13) Reciprocal Load Method

- The column interaction surface previously discussed can be alternatively plotted as a function of \( P_n \) and eccentricities \( e_x = M_{ny}/P_n \) and \( e_y = M_{nx}/P_n \) to produce the failure surface \( S_1 \).

- The failure surface \( S_1 \) can be transformed into equivalent failure surface \( S_2 \) where \( e_x \) and \( e_y \) can be plotted against \( 1/P_n \) instead of \( P_n \).

- \( e_x = e_y = 0 \) corresponds to the inverse of the capacity of the column if it were concentrically loaded \( P_0 \) (plotted as point \( C \)).

- For \( e_y = 0 \) and any given value of \( e_x \), there is a load \( P_{ny0} \) (corresponding to moment \( M_{ny0} \)) that would result in failure. The reciprocal of this load is plotted as point \( A \). Same goes for \( P_{nx0} \) of which the reciprocal is plotted as \( B \).

- The values \( P_{nx0} \) and \( P_{ny0} \) are easily established, for known eccentricities, using the methods already established for uniaxial bending or using design charts for uniaxial bending.
An oblique plane $S_2'$ is defined by three points: $A$, $B$, and $C$. This plane is used as an approximation of the actual failure surface $S_2$.

Note that, for any point on the surface $S_2$, there is a corresponding plane $S_2'$. Thus, the approximation of the true failure surface $S_2$ involves an infinite number of planes $S_2'$ determined by a particular pairs of values of $e_x$ and $e_y$, i.e. by particular points $A$, $B$, and $C$.

Observing the geometry of the approximating plane, it can be shown that:

$$\frac{1}{P_n} = \frac{1}{P_{nx0}} + \frac{1}{P_{ny0}} - \frac{1}{P_{n0}}$$  \hspace{1cm} (8.19)

Provided that $P_n \geq 0.10P_0$ and where:

$\to P_n$ = approximate value of nominal load in biaxial bending with eccentricities $e_x$ and $e_y$

$\to P_{ny0}$ = nominal load when only eccentricity $e_x$ is present ($e_y = 0$)

$\to P_{nx0}$ = nominal load when only eccentricity $e_y$ is present ($e_x = 0$)

$\to P_{n0}$ = nominal load for concentrically loaded column
To design a column using Reciprocal Load Method, do the following:

1. Select a trial column size and reinforcement.
2. Get the load eccentricities, $e_x$ and $e_y$.
3. Find the nominal loads $P_{nx0}$ and $P_{ny0}$ for uniaxial bending around the $X$ and $Y$ axes and $P_0$ for concentric loading using design charts.
4. Use equation 8.19 to calculate $1/P_n$.
5. Check that $\phi P_n \geq P_u$. 

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Example 3

The column shown is reinforced with eight No. 29 bars arranged around the column perimeter, providing an area \( A_s = 5160 \text{ mm}^2 \). A factored load \( P_u = 1100 \text{ kN} \) is to be applied with eccentricities \( e_y = 75 \text{ mm} \) and \( e_x = 150 \text{ mm} \). Material strengths are \( f_c' = 28 \text{ MPa} \) and \( f_y = 420 \text{ MPa} \). Check the adequacy of the design

a) Reciprocal load method.

b) Load contour method.